Input Admittance of a Coaxial-Line-Driven Cylindrical Cavity with a Center Dielectric Rod

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***Abstract*—An expression for the input admittance of a coaxially driven cylindrical cavity is presented. Two cases are presented: first, where the coaxial-line transition is located on one of the flat walls of an empty cavity, and secondly where there is a dielectric rod located coaxially between the two flat walls of the cavity. A comparison between the theory and measurements is presented which shows that the model is capable of yielding a high level of accuracy.**

***Index Terms*— Coaxial-line junctions, cylindrical cavities, dielectric permittivity properties, waveguide transitions.**

I. Introduction

JUNCTIONS between coaxial-line and waveguides have been widely considered (for example, [1] and [2]) and find application in a variety of microwave devices. The particular case where a dielectric material is located in a coaxially driven cavity has specific application in the area of permittivity measurement and microwave heating.

In this letter the case where a thin dielectric rod is located between the flat faces of a cylindrical cavity is considered (see Fig. 1). An accurate model which gives the input admittance of the junction between a coaxial-line and a variety of waveguide/cavity environments has been developed [3] and is applied here to find the admittance of a coaxial-line input located on one of the flat walls of the cavity.

A comparison between the theory and measurement is presented to demonstrate that the model may be used to optimize the coupling between the input coaxial-line and a rod of low dielectric permittivity.

II. Expression for the Input Admittance

Derivation of the input admittance of a coaxial-line (of inner conductor radius a and outer conductor radius b) feeding a radial-line waveguide (of height h) for the case where the center conductor of the coaxial-line extends the full height across the radial-line has been previously published [3], and so only the results will be given here. The input admittance can be shown to be given by

$$Y = \frac{1}{2h}\left\{\mathcal{Y}(0) + 2\sum_{m=1}^{\infty}\mathcal{Y}(m\pi/h)\right\} \qquad (1)$$

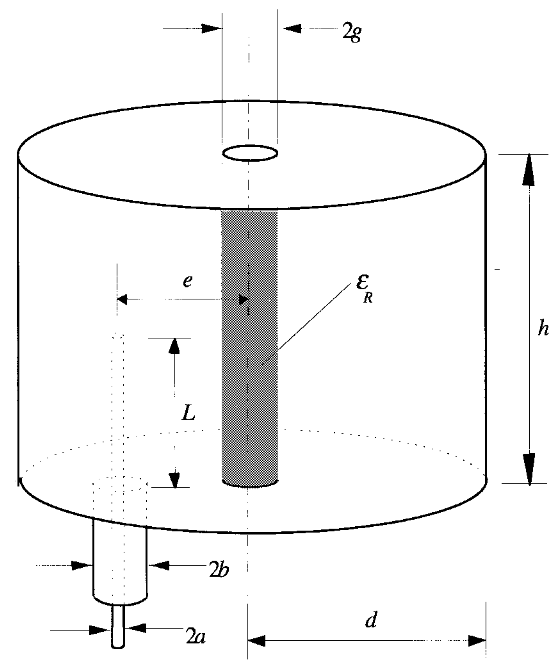

Fig. 1. Coaxially driven cylindrical cavity with a center dielectric rod.

where

$$\mathcal{Y}(\alpha) = \frac{j4\pi k_0}{q^2\eta_0 \ln^2(b/a)}\left\{\ln(b/a) + (I_0(qa)K_0(qb) - I_0(qb)K_0(qa))\frac{K_0(qb)+I_0(qb)S(\alpha)}{K_0(qa)+I_0(a)S(\alpha)}\right\} \qquad (2)$$

and where $q = \sqrt{\alpha^2 - k_0^2}, \eta_0$ is the intrinsic impedance of free space (377 Ω), k_0 is the free-space wavenumber, and $I_0(\cdot), K_0(\cdot), I_1(\cdot)$, and $K_1(\cdot)$ are zero- and first-order modified Bessel functions of the first and second kind, respectively. It should be pointed out that the modified Bessel functions used above should be replaced by their corresponding Bessel and Hankel functions when their arguments are imaginary.

It is relatively straightforward to extend (1) and (2) to the cases where: 1) the center conductor of the coaxial-line in the cavity is surrounded by a dielectric sheath [4]; 2) the center conductor only extends part of the way into the cavity [5]; and 3) the center conductor spans the full width of the cavity and has an arbitrary gap [6].

In order to find the solution for any type of region outside the coaxial-line $R > b$, all that is required is the "environment factor" $S(\alpha)$. This factor is the average, with respect to angular variation, of the spatial Fourier transform (FT) with respect to

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the z axis of the inward-traveling wave normalized to $I_0(qb)$ caused by the environment external to the coaxial-line junction when an axially symmetric outward traveling wave is assumed to be present (Fourier-transformed quantities are henceforth given in script). For example, for the case of a radial-line where there is no inward-traveling wave, the environment factor is $S(\alpha) = 0$.

The fields inside the coaxial-line junction region are assumed to be axially symmetric, but the fields outside the junction are not required to be axially symmetric. Environment factors for two particular cases have been derived in the past, viz. for a coaxially driven rectangular waveguide [7] and for an empty cylindrical cavity of radius $r = d$ with the coaxial-line located in the center $(e = 0)$ of the flat wall of the cavity [4].

III. Environment Factor for the Offset-Driven Cavity

Consider a cylindrical cavity of radius $r = d$ and base height $z = h$ which is excited by a coaxial-line port located at a distance $r = e$ from the center (see Fig. 1). Consider first the case where the cavity is empty except for the coaxial-line port. The FT of the outward radiated electric field from the coaxial-line junction is assumed to be axially symmetric and given by

$$\mathcal{E}_z^{\text{out}}(R, \varphi, \alpha) = K_0(qR) \tag{3}$$

where R is the radius relative to the center of the coaxial-line junction. While it is assumed that the junction is radiating axially symmetrically outwards relative to its own center, the reradiated fields reflected from the cavity wall are not axially symmetric and so the total field within the cavity will have θ dependence. Therefore it is advantageous at this point to express (3) relative to the center of the cavity in terms of r and θ using a well-known addition theorem for Bessel functions [8], i.e.,[1]

$$\mathcal{E}_z^{\text{out}}(r, \theta, \alpha) = \sum_{n=-\infty}^{\infty} K_n(qr) I_n(qe) \cos n\theta \tag{4}$$

where $I_n(\cdot)$ and $K_n(\cdot)$ are nth-order modified Bessel functions of the first and second kind. The (FT) electric field reflected by the cavity wall may be given by

$$\mathcal{E}_z^{\text{wall}}(r, \theta, \alpha) = \sum_{n=-\infty}^{\infty} B_n I_n(qr) \cos n\theta \tag{5}$$

where B_n are a set of coefficients to be found by applying the boundary condition that the total tangential (FT) electric field on the cavity wall is zero. By consideration of (4) and (5) and solving for the B_n coefficients, it can be shown that the total (FT) tangential electric field relative to the center of the cavity is given by

$$\mathcal{E}_z^{\text{total}}(r, \theta, \alpha) = \sum_{n=-\infty}^{\infty} I_n(qe) \left\{ K_n(qr) - I_n(qr) \frac{K_n(qd)}{I_n(qd)} \right\} \cos n\theta. \tag{6}$$

[1] This expression is only valid for $r > e$, but since it will only be used to evaluate the field at the cavity wall $r = d$, its use here is acceptable.

This expression for the total (FT) electric field is now be expressed relative to the center of the coaxial-line junction using another well-known addition theorem for Bessel functions [8], i.e.,[2]

$$\mathcal{E}_z^{\text{total}}(R, \varphi, \alpha) = K_n(qR) - \sum_{p=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} I_n(qR) I_{n+p}(qe) I_n(qe) \cdot \frac{K_n(qd)}{I_n(qd)} \cos p\varphi. \tag{7}$$

In order to find the environment factor it is necessary to find the average total (FT) electric field just external to the junction at $R = b^+$, with respect to angular variation around the coaxial-line junction. It is straightforward to show that the environment factor, $S(\alpha)$, for the off-center coaxial-line junction in an empty cylindrical cavity is given by

$$S(\alpha) = -\sum_{n=-\infty}^{\infty} I_n^2(qe) \frac{K_n(qd)}{I_n(qd)} \tag{8}$$

Note that (8) reduces to that given in [4] for $e = 0$.

Now consider the case where there is a dielectric rod of radius $r = g$ (it is assumed that $g \ll d$) and complex dielectric permittivity ε_R located at the center of the cavity extending the full height between the top and bottom flat walls of the cavity. If the additional boundary conditions that the tangential (FT) electric and magnetic fields are continuous at the boundary of the rod $r = g$ is applied then a new set of B_n coefficients may be found. If the condition is applied that the dielectric rod radius is electrically small, i.e., $g \ll \lambda$, then it may be assumed that the field scattered by the rod and produced inside the rod may be approximated as axially symmetric (with respect to the center of the cavity) and so only the B_0 coefficient is influenced by the rod. Thus it can be shown that the environment factor $S(\alpha)$ for the rod case is given by (8) with the $n = 0$ term of the summation replaced by

$$S_0(\alpha) = -I_0^2(qe) \frac{K_0(qd)}{I_0(qd)} \left[\frac{\dfrac{X_1}{X_2} + \dfrac{K_0(qe)}{I_0(qe)}}{\dfrac{X_1}{X_2} + \dfrac{K_0(qd)}{I_0(qd)}} \right] + K_0^2(qe) \frac{K_0(qd)}{I_0(qd)} \left[\frac{\dfrac{I_0(qd)}{K_0(qd)} - \dfrac{I_0(qe)}{K_0(qe)}}{\dfrac{X_1}{X_2} + \dfrac{K_0(qd)}{I_0(qd)}} \right] \tag{9}$$

where

$$X_1 = q_R I_0(q_R g) K_1(qg) - q\varepsilon_R I_1(q_R g) K_0(qg)$$
$$X_2 = q_R I_0(q_R g) I_1(qg) - q\varepsilon_R I_1(q_R g) I_0(qg)$$

and where $k_R (= k\sqrt{\varepsilon_R})$ is the wavenumber inside the dielectric rod and $q_R = \sqrt{\alpha^2 - k_R^2}$.

[2] This expression is only valid for $R < e$, but since it will only be used to evaluate the field near the coaxial-line junction, $R = b^+$, its use here is acceptable.

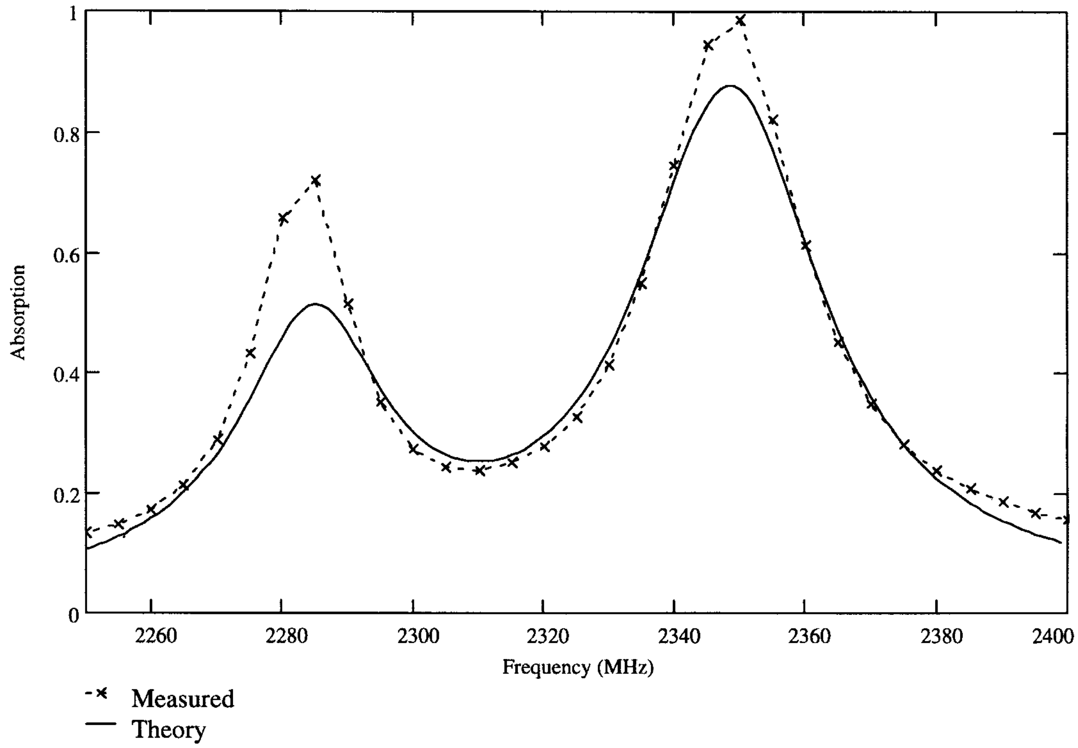

Fig. 2. Comparison of theory with measurements for a dielectric rod of radius 8 mm and relative permittivity $1.5 - j0.02$.

IV. Comparison Between Experimental and Theoretical Results

Consider a cylindrical cavity of dimensions $h = 296$ mm, $d = 48$ mm with a dielectric rod of radius $g = 8$ mm, and complex dielectric permittivity $\varepsilon_R = 1.5 - j0.02$. It is assumed that the cavity is excited by a 50-Ω PTFE-filled coaxial-line of dimensions a $= 0.607$ mm and b $= 2$ mm located at a distance $e = 35$ mm from the center of the cavity, where the inner conductor protrudes a distance $L = 24$ mm into the cavity. Fig. 2 shows the absorption, defined as $T = \sqrt{1 - |\Gamma|^2}$, where Γ is the input reflection coefficient calculated from the admittance given by (1), as a function of frequency (for a lossless dielectric rod or an empty cavity, $T = 0$ at all frequencies). Clearly the agreement between the theoretical and experimental results is good, however the lower absorption predicted by the model may be due in part to error in the estimation of the permittivity of the dielectric rod.

V. Conclusion

An expression for the input admittance of a coaxially driven cylindrical cavity has been given for the case where the coaxial-line is located on one on the flat walls of the cavity and offset from the center and where there is a dielectric rod located coaxially in the cavity between the two flat walls. A comparison between the theory and measurements shows that the model is capable of yielding a high level of accuracy.

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